

Hong Kong Baptist University
Faculty of Science
Department of Mathematics

Title (Units): **MATH 3680 APPLIED FUNCTIONAL ANALYSIS (3,3,0)**

approved by Sci. Board at Oct.18,2011

Course Aims: This course aims at familiarizing the student with the basic concepts, principles and methods of functional analysis and its applications. Functional analysis plays an important role in the applied sciences as well as in mathematics itself. Roughly speaking, functional analysis develops the tools from calculus and linear algebra further to the more general setting where one has vector spaces comprising functions or general abstract infinite-dimensional vector spaces. Problems from various application areas can then be conveniently posed in this common general set up, and solved using the techniques of functional analysis. The basic objects studied in functional analysis are vector spaces with a notion of distance between vectors, and continuous maps between such vector spaces. This interplay between the algebraic and analytic setting gives rise to many interesting and useful results, which have a wide range of applicability to diverse mathematical problems, such as from numerical analysis, differential and integral equations, optimization and approximation theory.

Prerequisite: MATH 1111-2 Mathematical Analysis I & II, MATH 1120 Linear Algebra, and MATH 2130 Real Analysis or Consent of instructor

Prepared by: Michael Ng

Learning Outcomes (LOs):

Upon successful completion of this course, students should be:

No.	Learning Outcomes (LOs)
	Knowledge
1	Able to understand the concept and theory of normed linear spaces and Banach spaces
2	Able to understand the concept and theory of continuity and bounded linear operators
3	Able to understand the theory of differentiation and an application in optimization
4	Able to understand the concept and theory of inner product spaces and Hilbert spaces
5	Able to understand the concept and theory of compact operators
	Skill
6	Able to apply the theory to solve mathematical problems
	Attitude
7	Able to solve problems independently

Assessment:

No.	Assessment Methods	Weighting	Remarks
1	Continuous Assessment (assignments, tests)	30%	Assignments are designed to measure students understanding of the theory, techniques, applications of functional analysis, the test is conducted to measure students' skills in solving functional analysis problems.
2	Final Examination	70%	Final Examination is designed to see how far students have achieved their intended learning outcomes especially in the Knowledge domain. Students should have a thorough understanding of the knowledge and apply them correctly in different context to do well in the exam.

Learning Outcomes and Weighting:

Content	LO No.	Teaching (in hours)
I. Normed linear spaces and Banach spaces	1,7	6
II. Continuity and Bounded Linear Operators	2,6,7	6
III. Differentiation and Optimization	3,6,7	3
IV. Inner Product Spaces and Hilbert spaces	4,6,7	15
V. Compact operators	5,6,7	10

Textbook: E. Kreyszig. Introductory Functional Analysis with Applications. Wiley, 1989

References: D.G. Luenberger. Optimization by Vector Space Methods. Wiley, 1969

W. Rudin. Real and Complex Analysis; 3rd edition. McGraw Hill, 1987.

Course Content in Outline:

<u>Topic</u>	<u>Hours</u>
I. Normed Linear Spaces and Banach Spaces	6
II. Continuity and Bounded Linear Operators	6
A. Linear transformations	
B. Continuous maps	
C. The Neumann series	
D. Left and right inverses	
III. Differentiation and Optimization	3
IV. Inner Product Spaces and Hilbert Spaces	15
A. Inner product spaces	
B. Orthogonal sets	
C. Best approximation in a subspace	
D. Fourier series	
E. Riesz representation theorem	
F. Adjoints of bounded operators	
V. Compact Operators	10
A. Approximation of compact operators	
B. Bolzano-Weierstrass theorem	
C. Uniform boundedness principle	